Numerical Simulation of Fluid-Structure Interaction for a Simplified Model of the Soft Palate

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  - Governing Equation
  - Computational Model
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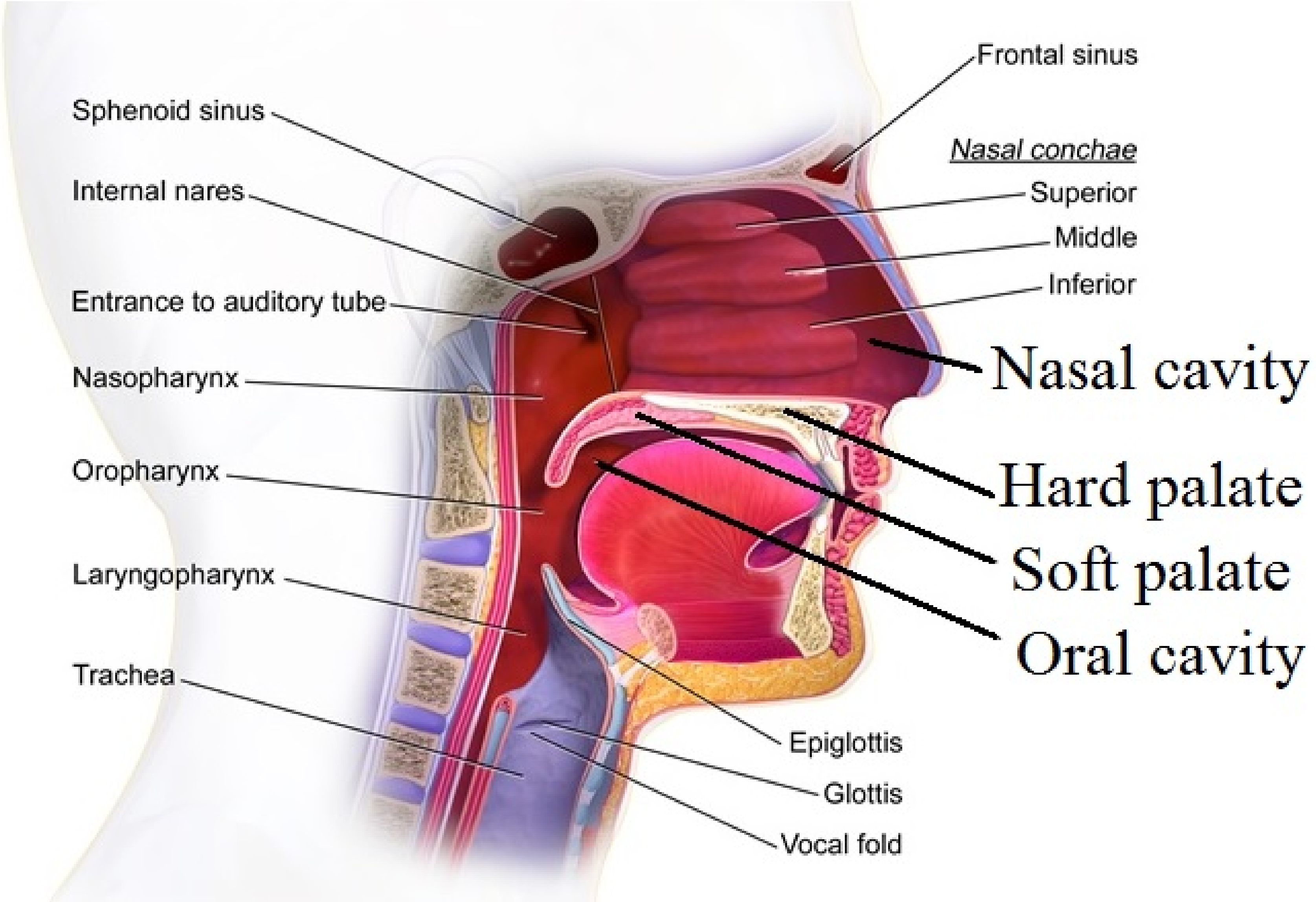
Motivation

- The current study is motivated by the intention to further understand upper airway dynamics aiming to explore computational modelling as a tool in diagnosis and treatment of Obstructive Sleep Apnea Syndrome (OSAS).

- OSAS affects an estimated 2-4% of the adults and about 10% of snorers being at risk of obstructive sleep apnea.

Anatomy of upper airway

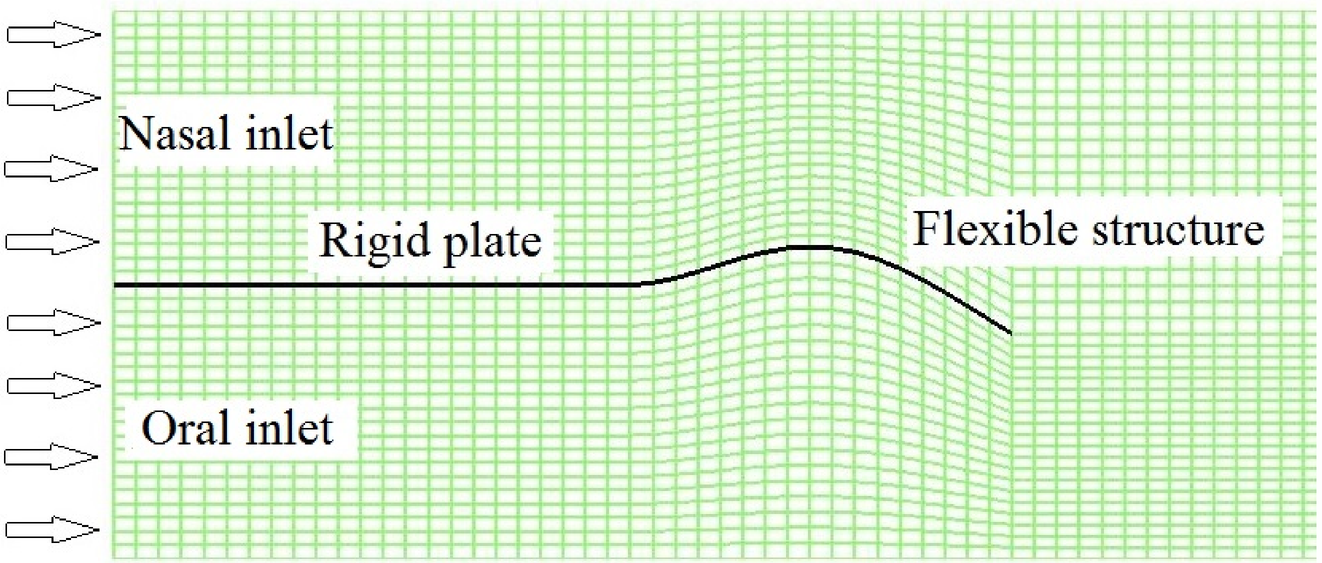


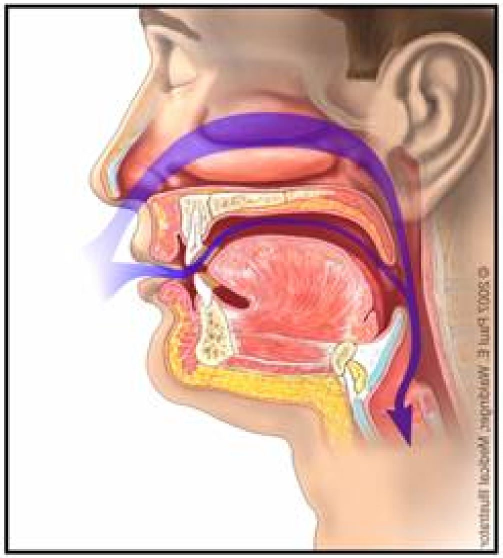
OSAS / Snoring

You can find the difference between Snoring and Obstructive Sleep Apnea in below link.

https://www.youtube.com/watch?v=inmop4Kv8PI

Assumptions





- Two dimensional (2D)
- Air: Perfect gas, Newtonian fluid, Compressible laminar flow
- Soft Palate: Cantilevered flexible plate, Thin-plate mechanics

Computational Model

- **Flow**
  - Compressible flow
  - Very low Mach number
  - Multi-block structured grid approach
  - The exchange of data between neighboring blocks is achieved by Message Passing Interface (MPI)

- **Structure**
  - Thin plate mechanics
  - Pressure driven

- **Interaction**
  - ALE formulation
  - Two-way coupled model

Fluid

- Compressible Navier-Stokes equations

Conservation law form

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot F^{C}(\mathbf{U}) = \nabla \cdot F^{V}(\mathbf{U}, \nabla \mathbf{U}) \qquad (1)$$

Conservative variables $\mathbf{U} = (\rho, \rho\mathbf{u}, \rho E)^{T}$

Flux vectors

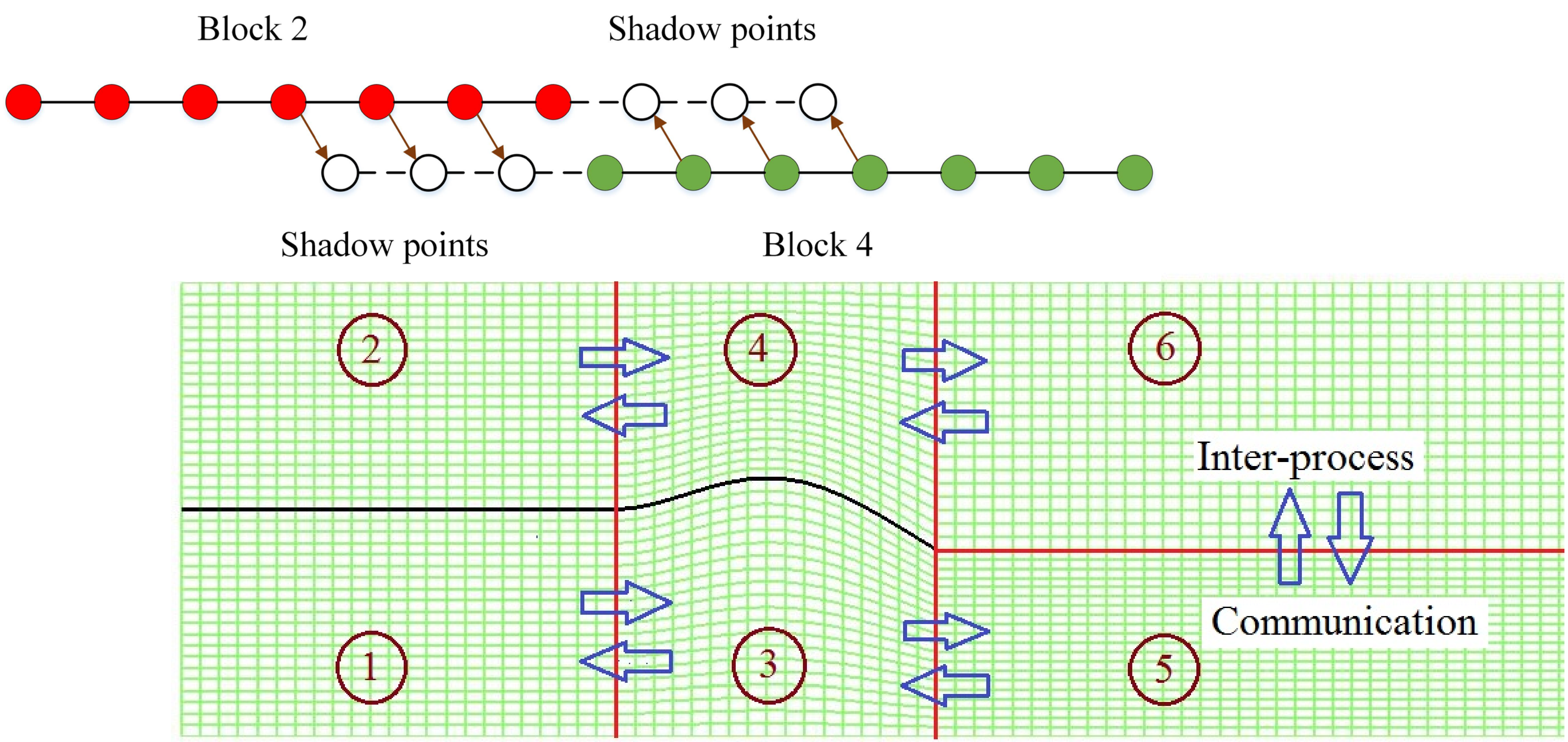
$$F^{C}(\mathbf{U}) = \begin{pmatrix} \rho\mathbf{u} \\ \rho\mathbf{u}\mathbf{u} + p\mathbf{I} \\ \rho H\mathbf{u} \end{pmatrix}, \quad F^{V}(\mathbf{U}, \nabla \mathbf{U}) = \begin{pmatrix} 0 \\ \tau \\ \tau \cdot \mathbf{u} + \kappa \nabla T \end{pmatrix} \qquad (2)$$

Computational Model

- Equations solved in a non-dimensional perturbation form.
- 4th order explicit Runge-Kutta method in time.
- 6th order explicit filter applied at the end of each time step to suppress undamped modes.

Computational Model

- 6^{th} order finite difference discretization in space with summation by parts (SBP) operators.



Structure

Motion of the elastic plate is modelled by classical thin-plate mechanics.

$$M\ddot{d} + C\dot{d} + B\,d_{xxxx} = -\Delta p \qquad (3)$$

Displacement *d* is constrained to be vertical.

Plate's mass $M = \rho_s \cdot h = (density \cdot thickness\ of\ plate)$

Damping C

Flexural rigidity $B = \frac{Eh^3}{12(1-\upsilon^2)}$

E, elastic modulus

υ, Poisson ratio

External force: fluid pressure $\Delta p = (p_{upper} - p_{lower})$

Computational Model

- Newmark time integration method is employed for solving implicit transient dynamics in the finite difference discretization.

$$\dot{d}_{t+\Delta t} = \dot{d}_t + \left[(1-\gamma)\ddot{d}_t + \gamma \ddot{d}_{t+\Delta t} \right] \Delta t \qquad (4)$$

$$d_{t+\Delta t} = d_t + \dot{d}_t \Delta t + \left[\left(\frac{1}{2} - \beta \right) \ddot{d}_t + \beta \ddot{d}_{t+\Delta t} \right] \Delta t^2 \qquad (5)$$

$\beta = 0.25$, $\gamma = 0.5$

- Central difference discretization is used for d_{xxxx} .

Computational Model

- Boundary conditions:
- Clamped configuration for the leading edge.

$$d_1 = \frac{\partial d_1}{\partial x} = 0$$

- Free end configuration for the trailing edge, i.e. bending moment and shear force is zero at the last node.

$$\frac{\partial^2 d_i}{\partial x^2} = \frac{\partial^3 d_i}{\partial x^3} = 0$$

Fluid-Structure Coupling

- ALE formulation for Navier-Stokes equations.

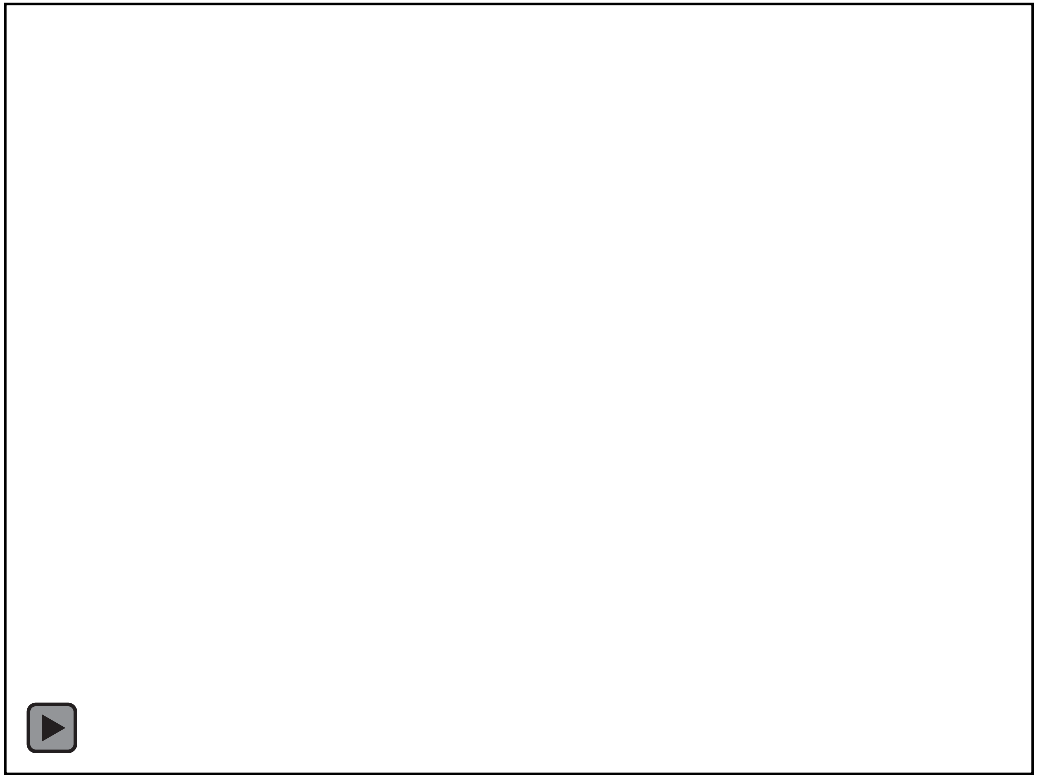
  The grid point velocity $\hat{\mathbf{u}}$ is subtracted from the material velocity $\mathbf{u}$ inside the convective derivative.

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla \xrightarrow{\textit{changed into}} \frac{D}{Dt} = \frac{\partial}{\partial t} + (\mathbf{u} - \hat{\mathbf{u}})\cdot\nabla$$

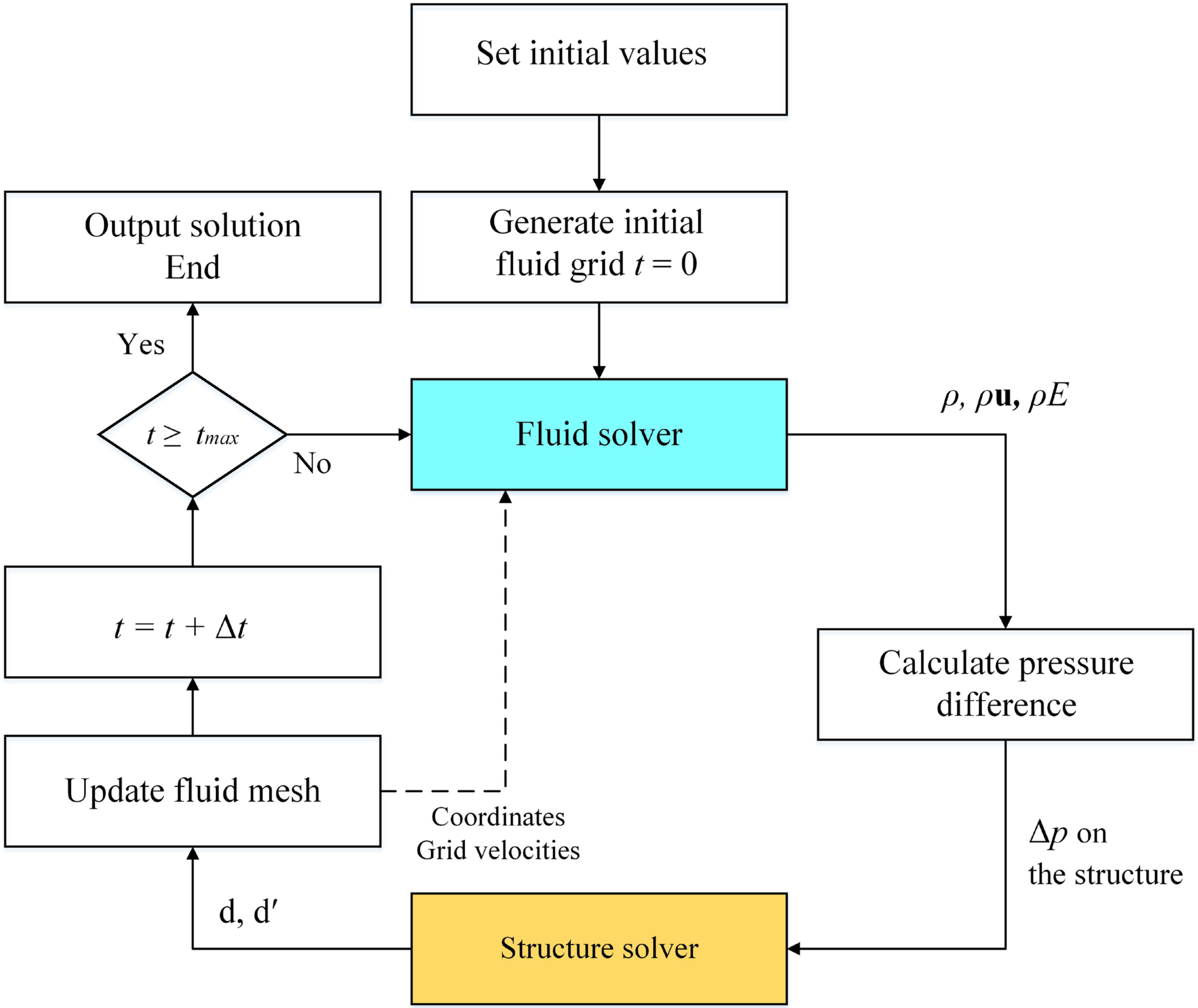
- Solving flow problems in a moving mesh described in an ALE framework needs the numerical scheme involved in the solver to obey the Geometric Conservation Law (GCL) for mathematical consistency.

Fluid-Structure Coupling

In mesh updating, the positions and velocities of the grid points in the fluid domain are linear interpolation of the positions and velocities of the structure.



Algorithm



Set initial values
Generate initial fluid grid $t = 0$
Output solution End
Yes
$t \geq t_{max}$
No
Fluid solver
ρ, $\rho\mathbf{u}$, ρE
$t = t + \Delta t$
Calculate pressure difference
Update fluid mesh
Coordinates Grid velocities
Δp on the structure
d, d′
Structure solver

Fluid-Structure Coupling

- The displacement and velocities are matched.

$$y_f = d_s$$

$$\dot{y}_f = \dot{d}_s$$

y_f and d_s represent the fluid grid and structural displacement at the interface. $\dot{y}_f$ and $\dot{d}_s$ are the grid velocity and structural velocity at the interface.

- The aerodynamic traction is simply the pressure acting on the plate surface.

$$\Delta p = \left(p_{upper} - p_{lower} \right) = f\left(external\ force\ on\ the\ plate \right)$$

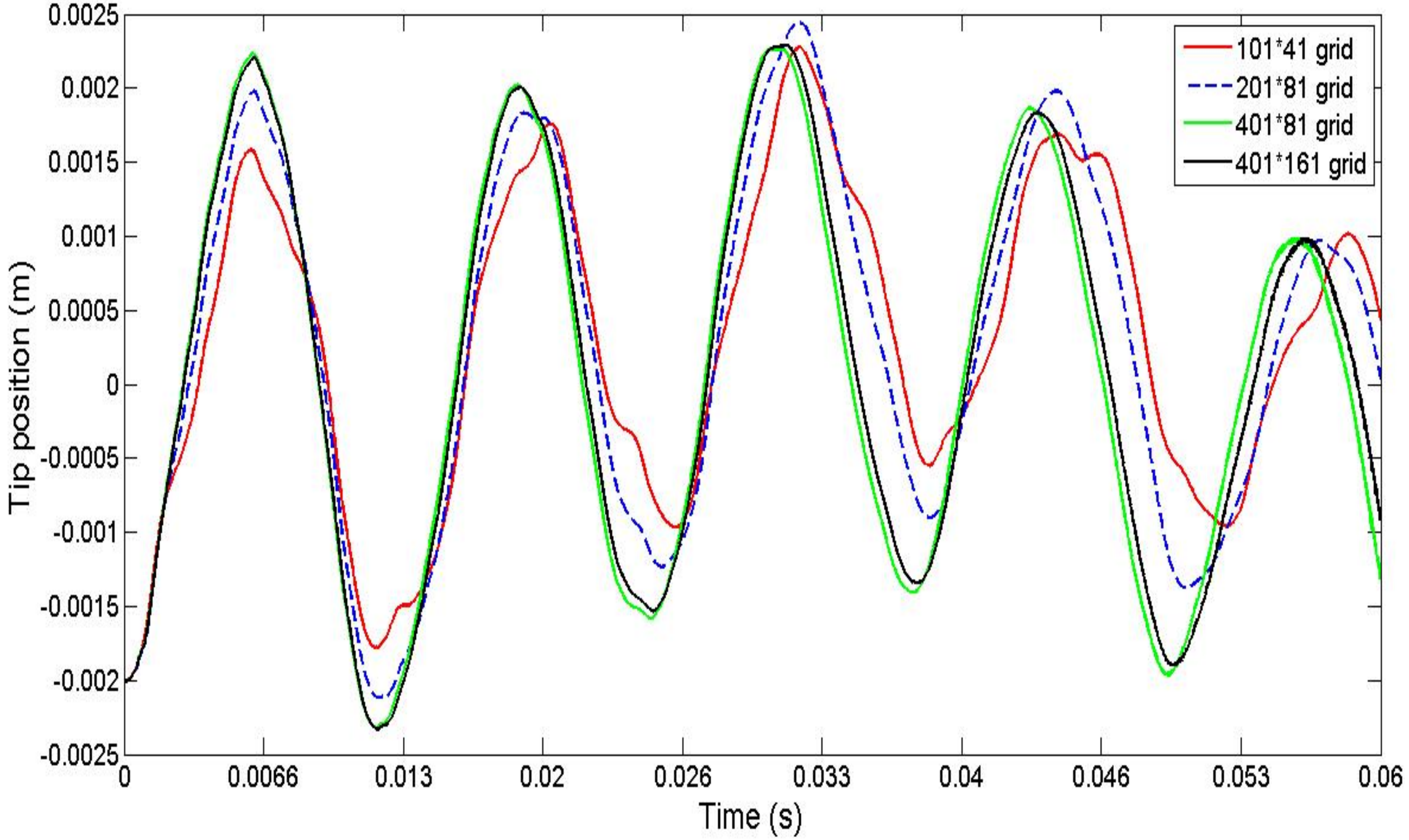
Results

An inlet velocity of 0.32 m/s is applied at Re = 378, which has been shown to promote cantilever plate instability in channel flow, corresponding to previous work in Balint & Lucey [2].

[2] Balint, T.S. & Lucey, A.D. 2005 Instability of a cantilevered flexible plate in viscous channel flow. Journal of Fluids and Structures 20, 893-912.

Results

The frequency obtained is between 75-91 Hz.



Results

The fluid velocity

Results

The fluid vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{u}$

Conclusions and Outlook

- Conclusions
  - ✓ It is found that the numerical model used here is suitable to investigate elastic deformation of the plate.
  - ✓ The numerical model is verified by previous work.
- Outlook
  - ✓ Validation is needed.
  - ✓ More realistic soft palate properties need to be applied for better correlation with clinically measured data.

End

- Acknowledgments

The current research is part of a larger research project entitled "Modeling of obstructive sleep apnea by fluid-structure interaction in the upper airways" which has been funded by the Research Council of Norway.

Thanks for your attention.

Appendix A

Conservative form in perturbation variables

$$\mathbf{U}'_t + \mathbf{F}'_x + \mathbf{G}'_y = 0$$

Flux vectors

$$\mathbf{F}' = \mathbf{F}^{C'} - \mathbf{F}^{V'}, \quad \mathbf{G}' = \mathbf{G}^{C'} + \mathbf{G}^{V'}$$

Solution vector

$$\mathbf{U}' = \left(\rho', (\rho\mathbf{u})', (\rho E)' \right)^T$$

Perturbation form

$$\rho = \rho - \rho_0, \ (\rho\mathbf{u})', \ (\rho E)' = \rho E - (\rho E)'$$